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ASYMPTOTIC PROPERTIES OF A STOCHASTIC DIFFUSION TRANSFER PROCESS WITH AN EQUILIBRIUM POINT OF A QUALITY CRITERION

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Abstract. *Weak convergence conditions are obtained for a diffusion transfer process with Markov switchings and a control with an equilibrium point of quality criterion functions. A procedure is constructed for the stochastic approximation of such a point in a scheme of series.*

Keywords: *stochastic diffusion equation, generator on a Banach space, Markov process, stochastic approximation procedure*.

INTRODUCTION

A random evolution in the form of a diffusion process with a control determined by the condition of reaching the extremum of a quality criterion function was studied in [1, 2]. A particular case is the existence of an equilibrium point of the quality criterion used in many applied optimal estimation problems [3, 4]. The problem of asymptotic behavior of systems under random disturbances [5] is independently considered. The latter problem was investigated using a small parameter in schemes of series and diffusion approximation [6]. To prove important statements, the Korolyuk model theorem [7] is used. In [8], a continuous procedure of stochastic optimization under the direct influence of a Markov process on regression functions and a pulse disturbance in a diffusion approximation scheme are considered.

This article considers the convergence of a diffusion transfer process with Markov switchings and a control with an equilibrium point of the quality criterion function for which a procedure of stochastic approximation in a scheme of series is constructed.

PROBLEM STATEMENT

Let a transfer process $y(t)\in\mathbf{R}^d$ be defined by the stochastic differential equation

$$dy(t)=a(y(t),x(t))dt+\sigma(y(t),x(t),u(t))dw(t), \tag{1}$$

where $x(t)$, $t>0$, is a uniformly ergodic Markov process defined by the generator

$$Q\varphi(x)=q(x)\int_X P(x,dy)[\varphi(y)-\varphi(x)] \tag{2}$$

in a measurable phase space (X,X) [6] on the Banach space $B(X)$ of real-valued bounded functions $\varphi(x)$ with the supremum norm

$$||\varphi(x)||=\sup_{x\in X}|\varphi(x)|.$$

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The generator Q is reducibly inverse on $B(X)$ with the projector $\Pi\varphi(x):=\int_X \pi(dx)\varphi(x)$, where $\pi(B)$ $(B\in \mathrm{X})$ is the stationary distribution of the Markov process $x(t)$, $t\geq 0$; this distribution is found from the relationships $\pi(dx)q(x)=q\rho(dx)$, $q=\int_X \pi(dx)q(x)$ ($\rho(dx)$ is the stationary distribution of the embedded Markov chain x_n, $n\geq 0$), and the potential R_0 of the Markov semigroup $R_0=\Pi-[Q+\Pi]^{-1}$.

The functions $a(y,x)=(a_k(y,x), k=\overline{1,d})$ and $\sigma(y,x,u)=(\sigma_k(y,x,u), k=\overline{1,d})$, $y\in \mathbf{R}^d$, $x\in X$, satisfy the conditions of existence of a global solution of the evolution equations

$$dy_x(t)=a(y_x(t),x)dt+\sigma(y_x(t),x,u_x(t))dw(t),\ \ x\in X, \tag{3}$$

for each fixed value of x of the Markov process $x(t)$, $t\geq 0$, in an interval $[\tau_i,\tau_{i+1}]$ of stay of the process $x(t)$, $t\geq 0$, at a state $x\in X$.

Let the quality criterion of transfer process (1) be defined by a function $G(y,x,u)$, $y\in \mathbf{R}^d$, with only one equilibrium point u_x^* on the interval $[\tau_i,\tau_{i+1}]$ that follows from the condition $G(y_x,x,u_x)=0$ or, in general representation (1), the control $u(t)$ is defined by the condition

$$G(y(t),x(t),u(t))=0. \tag{4}$$

Note that the solution of stochastic equation (1) on the interval $[\tau_i,\tau_{i+1}]$ under the nonrandom control $u(t)$ forms a Markov process.

To find asymptotic properties of the solution of problem (1), (4) in a scheme of series with a small parameter $\varepsilon>0$, we consider the stochastic equation

$$dy^\varepsilon(t)=a(y^\varepsilon(t),x(t/\varepsilon))dt+\sigma(y^\varepsilon(t),x(t/\varepsilon),u^\varepsilon(t))dw(t) \tag{5}$$

and the procedure of stochastic approximation

$$du^\varepsilon(t)=\alpha(t)G(y^\varepsilon(t),x(t/\varepsilon),u^\varepsilon(t))dt \tag{6}$$

with the common initial conditions

$$x(0)=x_0,\ \ y(0)=y_0,\ \ u(0)=u_0. \tag{7}$$

MAIN RESULT

THEOREM 1. Let $a(y,u)\in C(\mathbf{R}^d,\ \mathbf{R}^d)$, let $\sigma(y,x,u)\in C(\mathbf{R}^d,\ X,\mathbf{R}^d)$, and let $G(y,x,u)\in C(\mathbf{R}^d,\ X,\mathbf{R}^d)$.

Then, for an arbitrary ε ($\varepsilon<\varepsilon_0$ is sufficiently small), the following weak convergence takes place:

$$(y^\varepsilon(t),u^\varepsilon(t))\Rightarrow(\hat{y}(t),\hat{u}(t)), \tag{8}$$

where the limit process $(\hat{y}(t),\hat{u}(t))$ is specified by the generator

$$L\varphi(y,u)=A(y,u)\varphi(y,u)+\frac{1}{2}B(y,u)\varphi(y,x) \tag{9}$$

with the following representation in terms of test functions $\varphi(y,u)\in C^{3,2}(\mathbf{R}^d,\ \mathbf{R}^d)$:

$$A(y,u)=a(y)\varphi'_y(y,u)+\alpha(t)G(y,u)\varphi'_u(y,u), \tag{10}$$

where $a(y)=\int_X a(y,x)\pi(dx)$, $G(y,u)=\int_X G(y,x,u)\pi(dx)$, $B(u,y)=\hat{\sigma}^2(y,u)\varphi''_{yy}(y,u)$, and $\hat{\sigma}^2(y,u)=\int_X \sigma^2(y,x,u)\pi(dx)$.

COROLLARY 1. We will describe the limit control process $(\hat{y}(t), \hat{u}(t))$ by the equations

$$d\hat{y}(t) = a(\hat{y}(t))dt + \sigma(\hat{y}(t), \hat{u}(t))dw, \tag{11}$$

$$d\hat{u}(t) = \alpha(t)G(\hat{y}(t), \hat{u}(t))dt. \tag{12}$$

COROLLARY 2. Let us consider the transfer process described in a scheme of series by the stochastic differential equation

$$dy^{\varepsilon}(t) = a(y^{\varepsilon}(t), x(t/\varepsilon), u^{\varepsilon}(t))dt + \sigma(y^{\varepsilon}(t), x(t/\varepsilon), u^{\varepsilon}(t))dw(t)$$

with the control $u^{\varepsilon}(t)$ defined by the equation $du^{\varepsilon}(t) = \alpha(t)G(y^{\varepsilon}(t), x(t/\varepsilon), u^{\varepsilon}(t))dt$ and components $a(y,x,u), G(y,x,u), \sigma(y,x,u) \in C(\mathbf{R}^d, X, \mathbf{R}^d)$.

Then the weak convergence $(y^{\varepsilon}(t), u^{\varepsilon}(t)) \Rightarrow (\hat{y}(t), \hat{u}(t))$ takes place, where the limit process is defined over test functions $\varphi(y,x,u) \in C^{3,0,3}(\mathbf{R}^d, X, \mathbf{R}^d)$ by generator (9), where $A(u,y)\varphi(u,y) = a(y,u)\varphi'_y(y,x) + G(y,u)\varphi'_u(y,u)$, $a(y,u) = \int_X a(y,x,u)\pi(dx)$.

We first establish some properties of the generator of the three-component Markov process $y_t^{\varepsilon} = y_t^{\varepsilon}(t)$, $x_t^{\varepsilon} = x_t^{\varepsilon}(t)$, $u_t^{\varepsilon} = u_t^{\varepsilon}(t)$ defined by the relationship

$$L^{\varepsilon}(y,x)\varphi(y,x,u) = \lim_{\Delta \to \infty} \frac{1}{\Delta} E[\varphi(y_{t+\Delta t}^{\varepsilon}, x_{t+\Delta t}^{\varepsilon}, u_{t+\Delta t}^{\varepsilon}) - \varphi(y,x,u) \mid y_t^{\varepsilon} = y; x_t^{\varepsilon} = x; u_t^{\varepsilon} = u].$$

We introduce the following denotations for the conditional expectation with corresponding decompositions of increments:

$$E_{y,x,u}\varphi(y+\Delta y, x_{t+\Delta}^{\varepsilon}, u+\Delta u)$$
$$= E[\varphi(y+\Delta y, x_{t+\Delta}^{\varepsilon}, u+\Delta u) / y_t^{\varepsilon} = y; x_t^{\varepsilon} = x; u_t^{\varepsilon} = u].$$

Since

$$E_{y,x,u}\varphi(y+\Delta y, x_{t+\Delta}^{\varepsilon}, u+\Delta u)$$
$$= E_{y,x,u}\varphi\left(y + \int_t^{t+\Delta} a(y^{\varepsilon}(s),x)ds + \int_t^{t+\Delta} \sigma(y^{\varepsilon}(s),x,u^{\varepsilon}(s))dw(s), x, u+\Delta u\right) I(\theta > \varepsilon^{-1}\Delta)$$
$$+ E_{y,x,u}\varphi\left(u + \int_t^{t+\Delta} a(y^{\varepsilon}(s), x_{t+\Delta}^{\varepsilon})ds + \int_t^{t+\Delta} \sigma(y^{\varepsilon}(s), x_{t+\Delta}^{\varepsilon}, u^{\varepsilon}(s))dw(s), x_{t+\Delta}^{\varepsilon}, u+\Delta u)\right) I(\theta < \varepsilon^{-1}\Delta) + o(\Delta), \tag{13}$$

where θ is the sojourn time of the Markov process $x(t), t \geq 0$, at a state x, we have

$$I(\theta > \varepsilon^{-1}\Delta) = 1 - \varepsilon^{-1}q(x)\Delta + o(\Delta), \;\; I(\theta < \varepsilon^{-1}\Delta) = \varepsilon^{-1}q(x)\Delta + o(\Delta).$$

For the first addend in sum (13), we have

$$\varphi\left(y + \int_t^{t+\Delta} a(y^{\varepsilon}(s),x)ds + \int_t^{t+\Delta} \sigma(y^{\varepsilon}(s),x,u^{\varepsilon}(s))dw(s), x, u+\Delta u\right)$$
$$= \varphi\left(v + \int_t^{t+\Delta} \sigma(y^{\varepsilon}(s),x,u^{\varepsilon}(s))dw(s), x, u+\Delta u\right),$$

where $v = y + \int_t^{t+\Delta} a(y^{\varepsilon}(s),x)\,ds.$

For the latter representation of the test function with allowance for $\pm\varphi(v,x,u+\Delta u)$, we have

$$\varphi\left(v+\int_t^{t+\Delta}\sigma(y^\varepsilon(s),x,u^\varepsilon(s))dw(s),x,u+\Delta u\right)$$

$$=\varphi'_y(v,x,u+\Delta u)\int_t^{t+\Delta}\sigma(y^\varepsilon(s),x,u^\varepsilon(s))dw(s)$$

$$+\frac{1}{2}\varphi''_{yy}(v,x,u+\Delta u)\left[\int_t^{t+\Delta}\sigma(y^\varepsilon(s),x,u^\varepsilon(s))dw(s)\right]^2+\varphi(v,x,u+\Delta u)+o(\Delta). \quad (14)$$

Since

$$\varphi'_y(v,x,u+\Delta u)=\varphi'_y(v,x,u)+\varphi''_{uy}(v,x,y)\Delta u+o(\Delta)$$

$$=\varphi'_y(v,x,u)+\varphi''_{yu}(v,x,u)\alpha(t)G(y,x,u)\Delta+o(\Delta),$$

$$\varphi''_{yy}(v,x,u+\Delta u)=\varphi''_{yy}(v,x,u)+\varphi'''_{yyu}(v,x,u)\alpha(t)G(y,x,u)\Delta+o(\Delta),$$

for function (14), we obtain

$$\varphi\left(v+\int_t^{t+\Delta}\sigma(y^\varepsilon(s),x,u^\varepsilon(s))dw(s),x,u+\Delta u\right)$$

$$=\varphi(v,x,u)+\varphi'_u(v,x,u)\alpha(t)G(y,x,u)\Delta+o(\Delta)$$

$$+\varphi'_y(v,x,u)\int_t^{t+\Delta}\sigma(y^\varepsilon(s),x,u^\varepsilon(s))dw(s)$$

$$+\alpha(t)\varphi''_{yu}(v,x,u)G(y,x,u)\int_t^{t+\Delta}\sigma(y^\varepsilon(s),x,u^\varepsilon(s))dw(s)\Delta+o(\Delta)$$

$$+\frac{1}{2}\varphi''_{yy}(v,x,u)\left[\int_t^{t+\Delta}\sigma(y^\varepsilon(s),x,u^\varepsilon(s))dw(s)\right]^2$$

$$+\frac{1}{2}\alpha(t)\varphi'''_{yyu}(v,x,u)\left[\int_t^{t+\Delta}\sigma(y^\varepsilon(s),x,u^\varepsilon(s))dw(s)\right]^2 G(y,x,u)\Delta+o(\Delta). \quad (15)$$

Taking into account the representation of the variable v and continuous differentiability of test functions φ, we obtain

$$\varphi(v,x,u)=\varphi\left(y+\int_t^{t+\Delta}a(y^\varepsilon(s),x)ds,x\right)=\varphi(y,x,u)+\varphi'_y(y,x,u)a(y,x)\Delta+o(\Delta).$$

All the components with the variable v have similar representations in representation (15). Therefore, according to representation (15), we obtain

$$\varphi\left(v+\int_t^{t+\Delta}\sigma(y^\varepsilon(s),x,u^\varepsilon(s))dw(s),x,u+\Delta u\right)$$

$$=\varphi(y,x,u)+\varphi'_y(y,x,u)a(y,x)\Delta+o(\Delta)+\alpha(t)\varphi'_u(y,x,u)G(y,x,u)\Delta$$

$$+o(\Delta)+\varphi'_y(y,x,u)a(y,x)\int_t^{t+\Delta}\sigma(y^\varepsilon(s),x,u^\varepsilon(s))dw(s)\Delta+o(\Delta)$$

$$+\alpha(t)\varphi''_{yu}(y,x,u)G(y,x,u)\int_t^{t+\Delta}\sigma(y^\varepsilon(s),x,u^\varepsilon(s))dw(s)\Delta+o(\Delta)$$

$$+\frac{1}{2}\varphi''_{yy}(v,x,u)\left[\int_t^{t+\Delta}\sigma(y^\varepsilon(s),x,u^\varepsilon(s))dw(s)\right]^2+o(\Delta)$$

$$+\frac{1}{2}\alpha(t)\varphi'''_{yyu}(v,x,u)\left[\int_t^{t+\Delta}\sigma(y^\varepsilon(s),x,u^\varepsilon(s))dw(s)\right]^2 G(y,x,u)\Delta+o(\Delta).$$

Since the following relationships hold for conditional expectation:

$$E_{u,x,y}\int_t^{t+\Delta}\sigma(y^\varepsilon(s),x,u^\varepsilon(s))dw(s)=0,$$

$$E_{y,x,u}\left[\int_t^{t+\Delta}\sigma(y^\varepsilon(s),x,u^\varepsilon(s))dw(s)\right]^2=\sigma^2(y,x,u)\Delta+o(\Delta),$$

we obtain

$$E_{y,x,u}[\varphi(y+\Delta y,x^\varepsilon_{t+\Delta},u+\Delta u)]$$

$$=\varphi(y,x,u)+[\varphi'_y(y,x,u)a(y,x)+\alpha(t)\varphi'_u(y,x,u)G(y,x,u)]\Delta$$

$$+\frac{1}{2}\varphi''_{yy}(y,x,u)\sigma^2(y,x,u)\Delta-\varepsilon^{-1}q(x)E_{y,x,u}\varphi(y,x,u)\Delta$$

$$+\varepsilon^{-1}qE_{y,x,u}\varphi(y,x^\varepsilon_{t+\Delta},u)\Delta+o(\Delta).$$

Thus, for the generator $L^\varepsilon(y,x)$, we have

$$L^\varepsilon(y,x)\varphi(y,x,u)=\lim_{\Delta\to 0}\frac{1}{\Delta}\varepsilon^{-1}q(x)E_{y,x,u}[\varphi(y,x^\varepsilon_{t+\Delta},u)-\varphi(y,x,u)]$$

$$+\varphi'_y(y,x,u)a(y,x)+\alpha(t)\varphi'_u(y,x,u)G(y,x,u)+\frac{1}{2}\varphi''_{yy}(y,x,u)\sigma^2(y,x,u)$$

$$=\varepsilon^{-1}Q\varphi(y,x,u)+\varphi'_y(y,x,u)a(y,x)+\alpha(t)\varphi'_u(y,x,u)G(y,x,u)$$

$$+\frac{1}{2}\varphi''_{yy}(y,x,u)\sigma^2(y,x,u).$$

We will formulate the above reasoning in the form of the following statement.

LEMMA 1. The representation of the generator of the three-component Markov process

$$y^\varepsilon_t:=y^\varepsilon(t),\ x^\varepsilon_t:=x(t/\varepsilon),\ u^\varepsilon_t:=u^\varepsilon(t),\ t\ge 0,$$

in terms of test functions $\varphi(y,x,u)\in C^{3,0,2}(\mathbf{R}^d, X, \mathbf{R}^d)$ is as follows:

$$L^\varepsilon(y,x)\varphi(y,x,u)=\varepsilon^{-1}Q\varphi(y,x,u)+L(x)\varphi(y,x,u), \tag{16}$$

where

$$L(x)\varphi(y,x,u)=\varphi'_y(y,x,u)a(y,x)+\alpha(t)\varphi'_u(y,x,u)G(y,x,u)$$

$$+\frac{1}{2}\varphi''_{yy}(y,x,u)\sigma^2(y,x,u).$$

LEMMA 2. The solution of the problem of a singular perturbation for generator (16) in terms of test functions $\varphi^\varepsilon(y,x,u)=\varphi(y,u)+\varepsilon\varphi_1(y,x,u)$ defines the limit generator $L\varphi(y,u)=L_y\varphi(y,u)+L_u\varphi(y,u)$, where $L_y\varphi(y,u)=a(y)\varphi'_y(y,u)$ $+\frac{1}{2}\sigma^2(y,u)\varphi''_{yy}(y,u)$, $L_u\varphi(y,u)=\alpha(t)G(y,u)\varphi'_u(y,u)$, $a(y)=\int_X a(y,x)\pi(dx)$, $G(y,u)=\int_X G(y,x,u)\pi(dx)$, and $\sigma^2(y,u)$ $=\int_X \sigma^2(y,x,u)\pi(dx)$.

Proof. Consider the representation

$$L^\varepsilon(y,x)\varphi^\varepsilon(y,x,u)=\varepsilon^{-1}Q\varphi(y,u)+Q\varphi_1(y,x,u)+L(x)\varphi(y,u)+\varepsilon L(x)\varphi_1(y,x,u),$$

where

$$L(x)=\varphi'_y(y,u)\,a(y,x)+\alpha(t)G(y,x,u)\varphi'_u(y,u)+\frac{1}{2}\sigma^2(y,x,u)\varphi''_{yy}(y,u)$$

with the remainder in the form $\theta(x)=L(x)\varphi_1(y,x,u)=\theta_y(x)+\theta_u(x)$.

We write the expression $\varphi_1(y,x,u)$ in the form

$$\varphi_1(y,x,u)=R[L-L(x)]\varphi(y,u)=R_0\widetilde{L}(x)\varphi(y,u),$$

where

$$\widetilde{L}(x)=\widetilde{a}(y,x)\varphi'_y(y,u)+\alpha(t)\widetilde{G}(y,x,u)\varphi'_u(y,u)+\frac{1}{2}\widetilde{\sigma}^2(y,x,u)\varphi''_{yy}(y,u),$$

$$\widetilde{a}(y,x)=a(y)-a(y,x),\ \widetilde{G}(y,x,u)=G(y,u)-G(y,x,u),$$

$$\widetilde{\sigma}^2(y,x,u)=\sigma^2(y,u)-\sigma^2(y,x,u).$$

Thus, for the remainder terms $\theta_y(x)$ and $\theta_u(x)$, we have

$$\theta_y(x)=a(y,x)R_0[\widetilde{a}(y,x)\varphi'_y(y,u)]'_y+\frac{1}{2}a(y,x)R_0[\widetilde{\sigma}^2(y,x,u)\varphi''_{yy}]'_y$$

$$+\frac{1}{2}\sigma^2(y,x,u)R_0[\widetilde{a}(y,x)\varphi'_y(y,u)]''_{yu}+\frac{1}{4}\sigma^2(y,x,u)R_0[\widetilde{\sigma}^2(y,x,u)\varphi''_{yy}(y,u)]''_{yu},$$

$$\theta_u(x)=\alpha(t)a(y,x)R_0[\widetilde{G}(y,x,u)\varphi'_u(y,u)]'_u+\alpha(t)G(y,x,u)R_0[\widetilde{a}(y,x)\varphi'_y(y,u)]'_u$$

$$+\frac{1}{2}\alpha(t)G(y,x,u)R_0[\widetilde{\sigma}^2(y,x,u)\varphi''_{yu}(y,u)]'_u+\frac{1}{2}\alpha(t)\sigma^2(y,x,u)R_0[\widetilde{G}(y,x,u)\varphi'_u(y,u)]''_{yy}$$

$$+\alpha^2(t)G(y,x,u)R_0[\widetilde{G}(y,x,u)\varphi'_u(y,u)]'_u.$$

By the Korolyuk theorem [7], $L_y\varphi(y,u)$ defines a limit diffusion process that satisfies the equation $d\,\hat{y}(t)=a(\hat{y})dt+\sigma(\hat{y},\hat{u})dw(t)$ with the following control:

$$d\hat{u}=\alpha(t)G(\hat{y},\hat{u}(t))dt.$$

Proof of Theorem 1. The statement of Theorem 1 follows from the Korolyuk model theorem [7] and the result of Lemma 2.

THEOREM 2. Let the Lyapunov function $V(y,u)$ of the averaged system $\frac{\partial u}{\partial v}=G(y,u)$ be such that it satisfies the following conditions:

$$\text{Y1: } G(y,u)V'(y,u)<-cV(y,u),$$

$$\text{Y2: } |a(y,x)R_0[\widetilde{G}(y,x,u)V'_u(y,u)]'_y|\le c_1V(y,u),$$

$$|G(y,x,u)R_0[\widetilde{a}(y,x)V'_y(y,u)]'_u|\le c_2V(y,u),$$

$$|G(y,x,u)R_0[\widetilde{\sigma}^2(y,x,u)V''_{yu}(y,u)]'_u|\le c_3V(y,u),$$

$$|\sigma^2(y,x,u)R_0[\widetilde{G}(y,x,u)V'_u(y,u)]''_{yy}|\le c_4V(y,u),$$

$$|G(y,x,u)R_0[\widetilde{G}(y,x,u)V'_u(y,u)]'_u|\le c_5(1+V(y,u)).$$

Next, let the function $\alpha(t)$ be such that $\int_0^\infty \alpha(t)dt=\infty$, $\int_0^\infty \alpha^2(t)dt<\infty$. Then, for $\varepsilon>0$ and $\varepsilon\le\varepsilon_0$, where ε_0 is sufficiently small, the convergence $P\left\{\lim_{t\to\infty} u(t)=u^*\right\}=1$ takes place.

Proof. Consider now the generator of limit control $L_u^\varepsilon V(y,u)=L_uV(y,u)+\varepsilon\theta_u(x)$ for which we obtain the following estimate from conditions Y1 and Y2:

$$L_u^\varepsilon V(y,u)\le -c\alpha(t)V(y,u)+c^*\alpha^2(t)(1+V(y,u))$$

that implies the statement of Theorem 2 by the theorem considered in [3].

CONCLUSIONS

The asymptotic value of the control u^* allows one to consider fluctuations of a deviation of the control $u(t)$ from u^* and also to establish its basic characteristics.

REFERENCES

1. I. I. Gikhman and A. V. Skorokhod, Controlled Random Processes [in Russian], Naukova Dumka, Kiev (1977).
2. J. Jacod and A. N. Shiryaev, Limit theorems for Random Processes [Russian translation], Vol. 1, Fizmatgiz, Moscow (1994).
3. M. B. Nevelson and R. Z. Khasminskii, Stochastic Approximation and Recursive Estimation [in Russian], Nauka, Moscow (1972).
4. V. B. Kolmanovskii, “Problems of optimal estimation,” Soros Educational Journal, No. 11, 122–127 (1999).
5. A. V. Skorokhod, Asymptotic Methods in the Theory of Stochastic Differential Equations [in Russian], Naukova Dumka, Kiev (1987).
6. V. S. Korolyuk and N. Limnios, Stochastic Systems in Merging Phase Space, World Scientific, London–Singapore-Hong Kong (2005).
7. V. S. Korolyuk, Stochastic Models of Systems [in Ukrainian], Naukova Dumka, Kyiv (1989).
8. U. T. Khimka and Ya. M. Chabanyuk, “A difference stochastic optimization procedure with impulse perturbation,” Cybernetics and Systems Analysis, **49**, No. 3, 768–773 (2013).